Topic: Inverse trigonometric derivatives

Question: Find the derivative of the inverse trig function.

$$f(x) = \tan^{-1}\left(x^2 - 1\right)$$

Answer choices:

A $$f'(x) = \frac{2x}{x^4 - 2x^2 + 2}$$

B $$f'(x) = \frac{1}{x^4 - 2x^2 + 2}$$

C $$f'(x) = \frac{2x}{1 + 4x^2}$$

D $$f'(x) = \frac{1}{1 + 4x^2}$$

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Solution: A

First, note that $f(x)$ is a composite function. If we let $g(u) = \tan^{-1} u$ and $u = h(x) = x^2 - 1$, we'll find that $f(x) = g\left[h(x)\right]$.

Therefore, we'll need to use the chain rule to differentiate $f(x)$.

$$f(x) = g\left[h(x)\right]$$

$$f'(x) = g'\left[h(x)\right] \cdot h'(x)$$

Let's find the derivatives of g (using inverse trig derivative rules) and h.

$$g(u) = \tan^{-1} u$$

$$g'(u) = \frac{1}{1+u^2}$$

$$g'\left[h(x)\right] = \frac{1}{1+\left(x^2-1\right)^2} = \frac{1}{x^4-2x^2+2}$$

and

$$h(x) = x^2 - 1$$

$$h'(x) = 2x - 0 = 2x$$

Therefore,

$$f'(x) = g'\left[h(x)\right] \cdot h'(x)$$

$$f'(x) = \frac{1}{x^4-2x^2+2} \cdot 2x$$

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$$f'(x) = \frac{2x}{x^4 - 2x^2 + 2}$$

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Topic: Inverse trigonometric derivatives

Question: Find the derivative of the inverse trig function.

$$y = \frac{1}{\cos^{-1} x}$$

Answer choices:

A $\dfrac{dy}{dx} = \dfrac{1}{1 - x^2}$

B $\dfrac{dy}{dx} = \dfrac{1}{\left(\cos^{-1} x\right)^2 \sqrt{1 - x^2}}$

C $\dfrac{dy}{dx} = \dfrac{\sqrt{1 - x^2}}{-1}$

D $\dfrac{dy}{dx} = \dfrac{\sqrt{1 - x^2}}{\left(\cos^{-1} x\right)^2}$

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Solution: B

First, note that

$$y = \frac{1}{\cos^{-1} x}$$

can be rewritten as $y = \left(\cos^{-1} x\right)^{-1}$. In this notation, be careful that you don't confuse the two $^{-1}$ notations, as they mean different things. The first indicates that we're dealing with the inverse cosine function, and the second indicates that this function is being raised to the $^{-1}$ power.

From here, we can see that $f(x)$ is a composite function, and we can use the chain rule (along with the power rule) to differentiate it.

$$\frac{dy}{dx} = \frac{d}{dx}\left[\left(\cos^{-1} x\right)^{-1}\right]$$

$$\frac{dy}{dx} = -\left(\cos^{-1} x\right)^{-2} \cdot \frac{d}{dx}\left(\cos^{-1} x\right)$$

$$\frac{dy}{dx} = \frac{-1}{\left(\cos^{-1} x\right)^2} \cdot \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \frac{1}{\left(\cos^{-1} x\right)^2 \sqrt{1-x^2}}$$

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Topic: Inverse trigonometric derivatives

Question: Find the derivative of the inverse trig function.

$$f(x) = x \sin^{-1}\sqrt{x}$$

Answer choices:

A $$f'(x) = x \sin^{-1}\sqrt{x} + \frac{1}{\sqrt{1-x}}$$

B $$f'(x) = \sin^{-1}\sqrt{x} + \frac{x}{\sqrt{1-x}}$$

C $$f'(x) = x \sin^{-1}\sqrt{x} + \frac{1}{2\sqrt{x(1-x)}}$$

D $$f'(x) = \sin^{-1}\sqrt{x} + \sqrt{\frac{x}{4(1-x)}}$$

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Solution: D

First, note that $f(x) = x \sin^{-1}\sqrt{x}$ is the product of two functions of x. Therefore, we'll need to use the product rule to differentiate $f(x)$.

$$f(x) = g(x) \cdot h(x)$$

$$f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$$

$$f'(x) = 1 \cdot \sin^{-1}\sqrt{x} + x \cdot \frac{d}{dx}\sin^{-1}\sqrt{x}$$

Since $\sin^{-1}\sqrt{x}$ is a composite function, we'll need to use the chain rule to differentiate it (along with the power rule, and the rule for differentiating the inverse sine function).

$$\frac{d}{dx}\sin^{-1}\sqrt{x} = \frac{1}{\sqrt{1-\left(\sqrt{x}\right)^2}} \cdot \frac{d}{dx}\sqrt{x}$$

$$\frac{d}{dx}\sin^{-1}\sqrt{x} = \frac{1}{\sqrt{1-x}} \cdot \frac{d}{dx}x^{1/2}$$

$$\frac{d}{dx}\sin^{-1}\sqrt{x} = \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x(1-x)}}$$

Now we'll substitute back into our original product rule formula to continue finding $f'(x)$:

$$f'(x) = 1 \cdot \sin^{-1}\sqrt{x} + x \cdot \frac{1}{2\sqrt{x(1-x)}}$$

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$$f'(x) = 1 \cdot \sin^{-1}\sqrt{x} + \frac{x}{2\sqrt{x(1-x)}}$$

$$f'(x) = 1 \cdot \sin^{-1}\sqrt{x} + \frac{\sqrt{x}\sqrt{x}}{2\sqrt{x}\sqrt{(1-x)}}$$

$$f'(x) = \sin^{-1}\sqrt{x} + \frac{\sqrt{x}}{2\sqrt{1-x}}$$

$$f'(x) = \sin^{-1}\sqrt{x} + \frac{\sqrt{x}}{\sqrt{4}\sqrt{1-x}}$$

$$f'(x) = \sin^{-1}\sqrt{x} + \sqrt{\frac{x}{4(1-x)}}$$

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